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EFFECTS OF RANDOM BACKGROUND ON AUTOMATIC DETECTION IN LORAD.(U)

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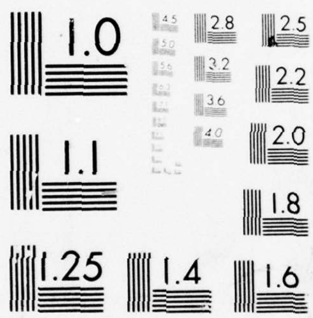

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6 EFFECTS OF RANDOM BACKGROUND ON AUTOMATIC DETECTION IN LORAD. (U)

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THE PROBLEM

Develop analytical techniques for treatment of target detection and classification processes. In particular, derive probability models required for analysis of automatic detection procedures in the Lorad system

RESULTS

1. The single-ping false alarm performance of the Lorad system has been analyzed. Formulas are presented which give the false alarm rates resulting from specified detection criteria. The treatment encompasses analog processes as well as computer programs.

2. A probability model has been developed which is applicable to a wide variety of search systems which process data in sampled form. In addition, these techniques are applicable to automatic target classification processes.

RECOMMENDATIONS

1. Apply the techniques to other sonar systems: SQS-23 modified for use with Small Ship Combat Data System (SSCDS); SQS-26.

2. Apply the methods to target classification processes in Lorad.

3. Develop detection probabilities for the Lorad system and couple these with false alarm characteristics derived in this report.

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ADMINISTRATIVE INFORMATION

Work was performed by members of the Human Factors Division under AS 02101, S-F001 03 02, Task 8016 (NEL E1-3) and S-F001 03 03, Task 8132 (NEL E1-11). This report covers work from March 1960 to September 1960 and was approved for publication 15 February 1961.

The analysis reported here was generated by discussions between the author and R. D. Isaak of the Acoustics Division concerning the characteristics of the information which must be handled by the USQ-20 computer in the Lorad system. G. P. Schumacher proposed, programmed, and analyzed the Monte Carlo experiments described in Appendix B. L. P. Mulcahy and J. A. Hammond performed the computer and manual calculations associated with the range-rate confidence level.

The author is indebted to R. D. Isaak, C. J. Van Vliet, and G. P. Schumacher for their critical review of the manuscript.

CONTENTS

Page	
4	INTRODUCTION
10	THEORY AND APPLICATIONS
10	Assumptions
10	The Poisson distribution
11	Number of events in TDS
14	Confidence-level probabilities
21	False alarm probability
22	False alarm rate
24	Remarks regarding interpretation
24	Control of false alarm rate
27	Time intervals between events
30	CONCLUSIONS
30	RECOMMENDATIONS
31	APPENDIX A: ALTERNATE PROCEDURE FOR ASSIGNING RRCL
33	APPENDIX B: MONTE CARLO EXPERIMENTS
40	APPENDIX C: GLOSSARY
42	REFERENCES

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TABLES

Page	Table	
8	1	Matrix of confidence level combinations
17	2	Spatial cluster probabilities
18	3	Bearing confidence level probabilities
19	4	Conditional probabilities $P(\text{RRCL} \mid k)$ for sequential procedure
20	5	Range-rate confidence level probabilities for sequential procedure
22	6	Matrix of joint probabilities $P(\text{BCL, RRCL})$ for sequential procedure
26	7	Bearing confidence level probabilities for alternate BCL criteria
26	8	Matrix of joint probabilities $P(\text{BCL, RRCL})$ for sequential procedure and alternate BCL criteria
26	9	P_{FA} and N_{FA} versus T threshold setting
32	10	Conditional probabilities $P(\text{RRCL} \mid k)$ for largest count procedure
32	11	Range-rate confidence level probabilities for largest count procedure
32	12	Matrix of joint probabilities $P(\text{BCL, RRCL})$ for largest count procedure

ILLUSTRATIONS

Page	Figure	
15	1	Probability of overflowing TDS storage list
16	2	Spatial cluster probabilities
29	3	Time-interval probabilities
36	4	Rayleigh probability density function
37	5	Tail of Rayleigh probability density function

INTRODUCTION

As part of the design of an operational noise-correlation Lorad system, a set of test programs for the AN/USQ-20 computer is being prepared to investigate the requirements for automatic detection (and ultimately classification) of possible target echoes.[1] (See list of references, p.42.) In the absence of substantial empirical data on the characteristics of random noise, reverberation, and submarine echoes at the output of such a noise-correlation system, the detection criteria being used are based on assumptions about the complex statistical properties of the background and signals. Many of the mathematical considerations applying to automatic detection have also entered into studies of Lorad display systems. Some theoretical investigations have therefore been undertaken in this area. The initial results, dealing with the effects of random backgrounds on automatic single-ping detection, are presented here.

In broad terms, the computer program to be considered examines the returns from a region of the ocean consisting of a 32° sector in bearing and three convergence zones in range. On a single-ping basis, this program attempts to detect and retain in memory possible submarine target echoes. These potential target echoes from a sequence of pings may then be presented on a visual display with which an operator may detect targets on the basis of track information. The following paragraphs will review the pertinent features of Lorad operation, describe in more detail the computer program of interest, and pose the theoretical questions which are to be discussed.

The processing of returns prior to entering data into the computer will be described first. The transmitted signal is a pulse with a nominal duration of 5 seconds, obtained by filtering a 100-c/s band from the output of a pseudo-random-noise generator. The pulse is transmitted in a 30° sector which is processed in the receiver in the form of sixteen

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[1] Reference 1 contains a more detailed description of Lorad and of the computer programs. Although much of the material in the Introduction is taken, with only minor modifications, from the reference, it is repeated here for the sake of completeness.

2° beams. The receiver is open for 70 seconds, corresponding to a total range coverage of 56,000 yards. Each beam is sampled every 5 milliseconds, or every 4 yards in range. The samples are stored in 16 signal deltics, each of which contains, say, 1000 samples corresponding to the transmitted pulse duration of 5 seconds. Each deltic is updated, then, every 5 milliseconds; i.e., the oldest sample is discarded and the new sample is added. During the 5-msec interval between samplings, the contents of all 16 signal deltics are correlated against the contents of eight range-rate reference deltics. (Actually, there are 24 references, since three frequency bands are employed to achieve a higher data rate for the system. However, this need not enter into the discussion here.) These references correspond to 6-knot range-rate intervals between minus 21 knots and plus 27 knots. Correlations are thus performed at the rate of 25,600 per second, and 1,792,000 correlations are performed per ping. The correlations are performed in a "series-parallel" fashion; the first beam is correlated with all eight references simultaneously, then the second beam, and so on.

Each correlator output is passed through a comb filter for signal-to-noise ratio improvement, and the teeth of the comb are recombined in an OR circuit.[2] The correlator outputs, which have a center frequency in the megacycle region as a result of the time compression processing, are then rectified and lowpass filtered to the band 0-200 c/s. The eight correlation values for a given beam are then OR-circuit processed so that only the largest value is retained. This reduces the data rate to 3200 correlation values per second, or 224,000 correlation values per ping. These correlation values are submitted to an amplitude threshold (analog), called "T," which provides the criterion for admitting data to the computer. A correlation value exceeding T is converted to digital form, assembled into a 30-bit word along with the appropriate range, bearing, and range-rate information, and is then entered into the TDS (temporary data storage) list in computer memory. Correlation values exceeding T, and sometimes the corresponding data words will hereafter be referred to as "events." The TDS list has a capacity of 4096. T is adjusted automatically by the computer to keep the average input rate such that the computer is always working near its maximum capability. This is roughly equivalent to filling TDS once per ping with T fixed to select, on the average, 1.8 per cent of the correlation values. (1.8 per cent of 224,000 is 4032, leaving a little leeway to avoid overflowing TDS. The overflow problem will be discussed later.) T will be assumed fixed (at the 1.8 per cent setting unless otherwise noted) throughout this report.

Prior to entry into TDS, correlation values are submitted to a second threshold (analog), called "*P*," which is set higher than *T*. *P* is the criterion for believing that a particular event which has exceeded *T* (and will thus be entered into TDS) should be singled out for further analysis. Events exceeding *P* are distinguished by having an indicator bit, which will be called the "*P*-bit," set to one. *P* is adjusted automatically by the computer and it is expected that one or two per cent of the words in TDS will have a *P*-bit of one.

The entering of data into the TDS list in the computer may be summarized as follows. On the average, 4032 events are entered into TDS per ping. An event may have a *P*-bit of zero or one, one of sixteen bearings, and one of eight range-rates. For the purposes of this report, it is assumed that the ranges entered are in 4-yard increments and are actual ranges of the returns being correlated. (Since the outer edge of the third zone occurs nominally at 300 seconds or 240,000 yards, there are 60,000 range increments and 16 bits would be required to enter the ranges. Actually, only 15 bits are used and the ranges are entered in 8-yard increments. This difference will not affect the analysis below and will therefore be neglected.) Since *T* is set to accept 1.8 per cent of the correlation values, one event is entered into TDS for each 56 correlation values, on the average. Range is a non-decreasing quantity from the beginning to the end of the list, though several events may be entered at the same range; however, the beams will generally be stepped through several times between events, and it is reasonable to expect that the bearings of the events in TDS will occur in essentially random order.

The computer program will now be described in greater detail. The basic concept underlying this program is the "cluster." In general, it is expected that a submarine target (and perhaps some nonsubmarine objects) will return a set of events which are grouped (clustered) in range, bearing and range-rate. This expectation is based on three assumptions: (1) that echoes from strong targets will often appear on two or more beams because of receiving beam overlap: (2) that an extended target such as a submarine will return echoes at several ranges, since correlations are performed every 4 yards; and (3) that a spatial cluster in range and bearing produced by a real target will exhibit more internal range-rate consistency than a cluster produced by random background. A preliminary perusal of the fairly large set of submarine echoes obtained with a limited noise-correlation system during the most recent trials with the USS BAYA (summer cruise, July and August 1960) appears to support

these assumptions. A detailed statistical analysis of these data is presently underway. The cluster concept is of considerable interest in relation to both detection and classification, and it is the *raison d'etre* of the analysis reported here. It should be kept in mind that the cluster, as it is discussed in this report, is a single-ping detection criterion.

The program proceeds as follows. The TDS list is searched until an event with a *P*-bit of one is found. This event (call it event *A*) is submitted to a third threshold (digital), called "*M*," which is set quite high. *M* is the criterion for retaining an event on the basis of correlation amplitude alone. This is necessary because a beam aspect target in the center of a receiving beam may produce only one or two events, but these are likely to be of particularly high amplitude. If event *A* exceeds *M*, all events in a range interval of about 100 yards and a bearing interval of four beams (called a "cluster-sized region" hereafter) about event *A* are stored as a cluster in the PTS (possible target storage) list in computer memory. The exact manner in which the 100-yard interval and the four beams are selected is not of importance here.

If event *A* does not exceed *M*, it is considered as a member of a possible cluster. The TDS list is searched to determine whether a set of events meeting the necessary cluster criteria is present in a cluster-sized region centered in some way about event *A*. If such a cluster is found, it is entered into PTS. If not, event *A* is discarded and TDS is searched for a new event with *P*-bit of one. Then the above procedure is repeated. After TDS has been completely searched in the manner described for a sequence of pings, PTS contains the events needed for the visual track-detection display mentioned earlier.

The present criteria for selecting a set of events as a cluster for entry into PTS will now be described. The first step is to set up the appropriate range and bearing limits around event *A*; i.e., to delimit the cluster-sized region to be studied. Then the number of events (including event *A*) in the region is counted. Four bearing confidence levels (BCL's) are assigned on the basis of this count as follows: If the count is 1,2, or 3, BCL 0 is assigned; if the count is 4, 5,or 6, BCL 1 is assigned; if the count is 7 or 8, BCL 2 is assigned; and if the count is 9 or greater, BCL 3 is assigned. The criteria given here for BCL assignment supersede, tentatively, those shown in figure 8 of reference 1. These two sets of criteria are compared in the section on "Control of False Alarm Rate," to follow. If BCL is 0, the

cluster is discarded. If BCL is 3, the cluster is entered into PTS without further tests.

If BCL is 1 or 2, the events are tested for range-rate consistency and a range-rate confidence level (RRCL) is assigned. At present, a sequential procedure for assigning RRCL is being considered and this procedure is the one that will be described and analyzed below. (An alternate procedure, which may well be adopted in the future, is described and analyzed in Appendix A.) First, a count is made of the number of events (including event A) having the same range-rate as event A. If the count is 1, a count is made of the number of events in the 6-knot range-rate increment above that of event A. If this count is 0 or 1, a count is made for the 6-knot increment below that of event A. If this final count is 0 or 1, RRCL 0 is assigned and the cluster is discarded. Now, suppose a count of 2 or greater is obtained during the procedure outline. In this case, the procedure is halted and a range-rate confidence level is assigned as follows: If the count is 2, RRCL 1 is assigned; if the count is 3, RRCL 2 is assigned; and if the count is 4 or greater, RRCL 3 is assigned. If RRCL is 3, the cluster is entered into PTS without further tests.

Thus far in the program, the following decisions have been made: (1) clusters with BCL 0 have been discarded; (2) clusters with BCL 3 have been entered into PTS; (3) clusters with BCL 1 or 2 and RRCL 0 have been discarded; and (4) clusters with BCL 1 or 2 and RRCL 3 have been entered into PTS. The clusters still to be sorted are those with BCL 1 or 2 and RRCL 1 or 2. The decisions for these are: (5) clusters whose confidence levels sum to less than 3 are discarded; and (6) clusters whose confidence levels sum to 3 or greater are entered into PTS. These six decisions have been presented roughly in the order they are made in the program. It is noted that decision (6) actually subsumes all the others, so that the selection of clusters may be described in terms of this single decision or criterion (BCL + RRCL $\geq$ 3).

The above discussion of confidence levels and decisions may be summarized with the aid of table 1.

Table 1. Matrix of Confidence Level Combinations.

		RRCL 0	RRCL 1	RRCL 2	RRCL 3
BCL	0	1	2	3	4
	1	5	6	7	8
	2	9	10	11	12
	3	13	14	15	16

Combination 4 is impossible. Decision (1) discards combinations 1, 2 and 3. Decision (2) accepts combinations 13 - 16. Decision (3) discards combinations 5 and 9. Decision (4) accepts combinations 8 and 12. Decisions (5) and (6) discard combination 6 and accept combinations 7, 10 and 11. The complete criterion BCL + RRCL $\geq$ 3, of course, accepts all possible combinations on or below the secondary diagonal of the matrix.

The central question raised by the cluster concept, assuming that the cluster model is appropriate to submarine echoes, is that of false alarm probability: How likely is it that a cluster meeting the criteria described above will be produced by noise, reverberation, or false targets? This question must eventually be answered with a reasonable degree of confidence if we are to specify computer information rates, threshold settings, and display techniques which are, in some sense, optimum.* This report makes an initial contribution toward that end by investigating in detail the effects of random background. It is felt that this analysis covers the effects of both random noise and random reverberation. Independent analysis pertinent specifically to reverberation has also been undertaken.[3]

The following topics will be discussed here:

1. The probability of observing exactly k events in a time interval t if the events occur at random instants in time. The results are applied to:

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* The companion question to be answered is, of course, that of detection probability: How likely is it that a submarine target which ventures within the ocean volume being searched by the Lorad system will be detected? The false alarm contributions of noise and reverberation are, for the most part, susceptible to analytical treatment. On the other hand, the primary questions associated with real and false targets must ultimately be answered on an empirical basis; they are intimately connected with the physical properties of the target, the medium, and the propagation path, as well as with the processing-gain of the Lorad system and the appropriateness of the detection model (the cluster model, in this case) being used. It is hoped that an analysis of the data collected during the recent sea trials mentioned earlier will provide tentative answers to some of these questions.

a. The probability distribution of the number of events in TDS. The probability of overflowing TDS.

b. The probability of a false alarm being produced by random noise at the correlator outputs. False alarm rates for the Lorad system.

2. The probability distribution of the time intervals between events which occur at random instants in time. The results are applied to the time intervals between events entering TDS.

These results are presented in the following section, Theory and Applications. Monte Carlo experiments verifying the basic probability distributions used below are described in Appendix B.

THEORY AND APPLICATIONS

1. ASSUMPTIONS

The starting point of this analysis is the sequence of correlation values presented to threshold T. These correlation values are considered to be produced as the result of random noise at the inputs of the signal deltics. It is assumed that the correlation values may then be treated as samples of a stationary random process. It is further assumed that consecutive correlation values (i.e., correlation values separated by one 4-yard range increment in a single beam) are independent. Although this latter assumption might be somewhat suspect since the true range resolution of the Lorad system is 8 yards rather than 4 yards, the assumption is justified for the purposes of this report. If the two assumptions stated above are accepted, along with a fixed (percentage) setting of t, the questions posed in the Introduction can be completely answered without any information concerning the amplitude distribution of the correlation values; therefore, this distribution will not be discussed except in Appendix B on the Monte Carlo experiments. This feature lends a considerable amount of generality to the analyses presented below.

2. THE POISSON DISTRIBUTION

Consider a sequence of events which occur at random times and

which obey the stationarity and independence assumptions stated above. The probability $P(k; \lambda t)$ of finding exactly k events in an interval of length t is given by the Poisson distribution[4]

$$P\ (k; \lambda t) = \frac{(\lambda t)^k}{k\ !}\ \exp\ (-\lambda t) \tag{1}$$

If S denotes the number of events observed, (1) gives the probability that $S = k$, a statement which will be written

$$Pr\ \{S = k\} = P\ (k; \lambda t) \tag{2}$$

The mean, second moment, and variance (second central moment) of the Poisson distribution are, respectively,[5]

$$\mu = \mu_1' = \lambda t \tag{3}$$

$$\mu_2' = \lambda t\ (\lambda t + 1) \tag{4}$$

$$\sigma^2 = \mu_2 = \mu_2' - \mu^2 = \lambda t \tag{5}$$

From (3), we see that λ is the mean number of events per unit time.

3. NUMBER OF EVENTS IN TDS

The Poisson distribution will be used to determine the probability distribution of the number of events entered into TDS for a single ping. The probability of overflowing TDS with a single ping will also be derived.

For convenience, we divide time into intervals of length t_0 (5 milliseconds) corresponding to the 4-yard range increments

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[4]For a thorough presentation of the Poisson distribution with numerous applications, see Chapter 6 of reference 4.

[5]The moment notations employed here are those used in Chapter 5 of reference 5.

between correlations. In a single beam, the number of events entered into TDS per interval t_0 is either 0 or 1. The mean number of events per beam per interval t_0 is 0.018 (since the T threshold is assumed set to select 1.8 per cent of the correlation values). From (3), then, the mean number of events per beam per unit time is

$$\lambda_0 = 0.018/t_0 \tag{6}$$

For the full 56,000 yards of range coverage, the total time interval is $t = 14{,}000\ t_0$ and $\lambda_0 t = 252$. Thus, the probability of entering exactly k events per ping into TDS from a single beam is

$$P(k;\ 252) = \frac{(252)^k}{k!} \exp(-\ 252) \tag{7}$$

For all 16 beams, the mean number of events per unit time is

$$\lambda_T = 16\lambda_0 = 0.288/t_0 \tag{8}$$

and $\lambda_T t = 4032$. Finally, the probability of entering exactly k events per ping into TDS is

$$P(k;\ 4032) = \frac{(4032)^k}{k!} \exp(-\ 4032) \tag{9}$$

From (3) and (5), the mean number of events per ping is 4032 and the standard deviation (σ) of the number of events per ping is $(4032)^{\frac{1}{2}} \sim 63.5$.

It is seen that the size 4096 chosen for TDS is approximately one standard deviation greater than the mean or expected number of events. The probability of overflowing TDS on a single ping is

$$Pr\{S \geq 4097\} = \sum_{k=4097}^{\infty} P(k;\ 4032). \tag{10}$$

Clearly, the direct evaluation of (10) would be quite difficult. Fortunately, the Poisson distribution may be approximated by a Gaussian (or normal) probability density function when the value of λ is large enough.*

The standard (zero mean and unity standard deviation) Gaussian density function is

$$\phi(x) = (2\pi)^{-\frac{1}{2}} \exp(-x^2/2) \qquad (11)$$

and the corresponding (cumulative) distribution function is

$$\Phi(x) = \int_{-\infty}^{x} \phi(y)\,dy \qquad (12)$$

For a Poisson process with parameter λ, where λ is large, we can make the following approximation: the probability that the number of events observed in an interval of length t will fall between a and b is

$$Pr\{a \le S \le b\} = \sum_{k=a}^{b} P(k;\lambda t)$$

$$\sim \Phi\left[\frac{b - \lambda t + \frac{1}{2}}{(\lambda t)^{\frac{1}{2}}}\right] - \Phi\left[\frac{a - \lambda t - \frac{1}{2}}{(\lambda t)^{\frac{1}{2}}}\right] \qquad (13)$$

In the case $b = \infty$, as in (10), (13) reduces to

$$Pr\{S \ge a\} \sim 1 - \Phi\left[\frac{a - \lambda t - \frac{1}{2}}{(\lambda t)^{\frac{1}{2}}}\right] \qquad (14)$$

Returning now to the overflow problem, (14) allows us to write the overflow probability in (10) in the form

$$Pr\{S \ge a\} \sim 1 - \Phi\left[\frac{a - 4032.5}{63.5}\right] \qquad (15)$$

*See ref. 4, Chapter 7

where a is one greater than the size assigned to TDS. Equation 15 gives the probability of overflowing a TDS list of size 4096 (a = 4097) as 0.155.

The effect of increasing the size of TDS (assuming the T threshold still fixed at the 1.8 per cent setting) on the probability of overflow is indicated in figure 1. A scale for T has been included at the right edge of this figure; this scale indicates the effect of decreasing T with the size of TDS fixed at 4096. Here, T is expressed in probability form (T = 0.018 corresponds to the 1.8 per cent setting).

4. CONFIDENCE LEVEL PROBABILITIES

In this section, we will develop various probabilities associated with the bearing and range-rate confidence levels of a cluster. These will be employed in the next section to determine the probability of a false alarm. We begin by calculating the probability distribution of the number of events in a cluster-sized region R having a range extent of 108 yards and a bearing extent of four 2° beams.

For 108 yards, $t = 27t_0$. For a single beam, equation 6 gives $\lambda_0 t = 0.486$. The probability of finding exactly k events in a 108-yard range interval for a single beam is

$$P(k;\ 0.486) = \frac{(0.486)^k}{k!} \exp(-\,0.486)$$

$$= 0.615\frac{(0.486)^k}{k!} \tag{16}$$

For four beams, $4\lambda_0 t = 1.944$ and the probability of finding exactly k events in R is

$$P(k;\ 1.944) = 0.143\ \frac{(1.944)^k}{k!} \tag{17}$$

Probabilities calculated using (16) and (17) are tabulated in the second and fourth columns of table 2 and are plotted using filled circles and triangles in figure 2. The Monte Carlo data will be discussed later.

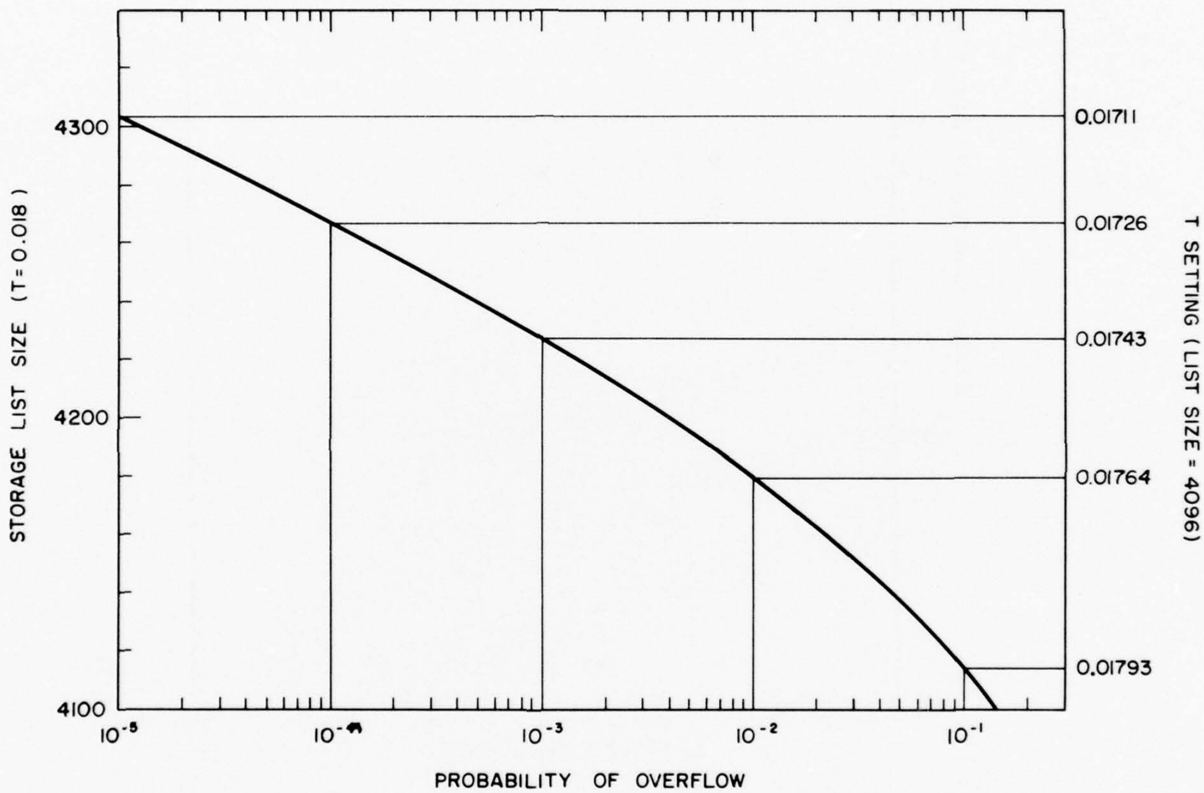

Figure 1. Probability of overflowing TDS storage list.

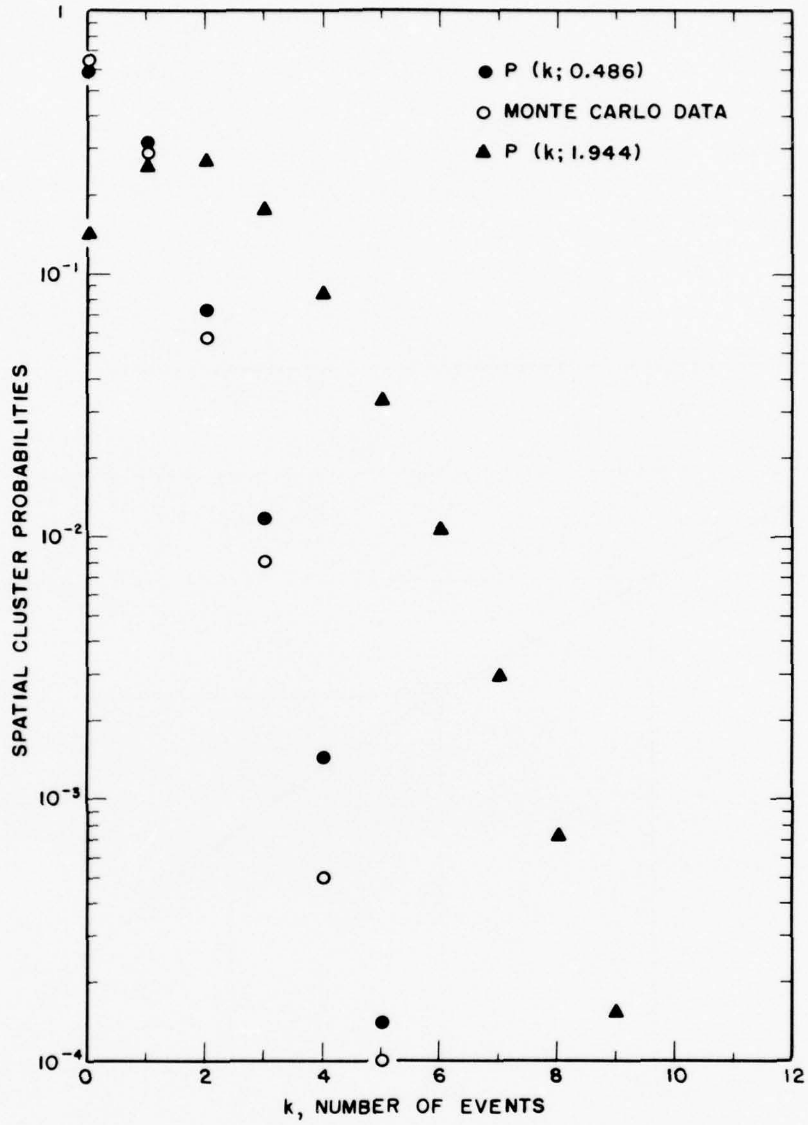

Figure 2. Spatial cluster probabilities.

Table 2. Spatial cluster probabilities.

k	$P(k;\ 0.486)$	Monte Carlo Data	$P(k;\ 1.944)$	$P(k)$
0	0.615	0.642	0.143	0
1	0.299	0.293	0.278	0.324
2	7.26×10^{-2}	5.68×10^{-2}	0.270	0.317
3	1.18×10^{-2}	0.8×10^{-2}	0.175	0.204
4	1.43×10^{-3}	0.5×10^{-3}	8.51×10^{-2}	9.93×10^{-2}
5	1.39×10^{-4}	1.0×10^{-4}	3.30×10^{-2}	3.85×10^{-2}
6	1.13×10^{-5}	0	1.07×10^{-2}	1.25×10^{-2}
7			2.97×10^{-3}	3.47×10^{-3}
8			7.24×10^{-4}	8.45×10^{-4}
9			1.56×10^{-4}	1.82×10^{-4}
10			3.03×10^{-5}	3.54×10^{-5}

The probabilities $P(k;\ 1.944)$ may be used to determine the probabilities of obtaining each of the 4 bearing confidence levels for a given cluster. However, $P(k;\ 1.944)$ cannot be used directly because a cluster must have at least one event in it (i. e., the $k = 0$ case does not occur). To account for this situation, we define a spatial cluster probability $P(k)$ as follows:

$$P(k) = \begin{cases} 0, \quad k = 0 \\ \left[\dfrac{1}{1 - P(0;\ 1.944)}\right] P(k;\ 1.944) \\ \quad = 1.167\ P(k;\ 1.944), \quad k \geq 1 \end{cases} \tag{18}$$

These probabilities are listed in the final column of table 2. The confidence level probabilities $P(\mathrm{BCL})$ are now given by sums of the $P(k)$'s

$$Pr\{\mathrm{BCL}=0\} = Pr\{k=1\} + Pr\{k=2\} + Pr\{k=3\} \tag{19}$$

$$Pr\{BCL=1\} = Pr\{k=4\} + Pr\{k=5\} + Pr\{k=6\} \quad (20)$$

$$Pr\{BCL=2\} = Pr\{k=7\} + Pr\{k=8\} \quad (21)$$

$$Pr\{BCL=3\} = Pr\{k \geq 9\} = 1 - \sum_{i=0}^{2} Pr\{BCL=i\} \quad (22)$$

and are presented in table 3.

Table 3. Bearing confidence level probabilities.

k	BCL	P(BCL)
1,2,3	0	0.845
4,5,6	1	0.1503
7,8	2	4.315×10^{-3}
≥ 9	3	3.85×10^{-4}

The next step is to compute the probabilities of the range-rate confidence levels. Since RRCL is a function of k, the number of events in the region R, we begin by determining the conditional probabilities $P(\text{RRCL}\,|\,k)$; i.e., the probability of RRCL given the value of k.

We recall that 1, 2 or 3 of the eight range-rate channels are examined in assigning RRCL. It will be assumed that an event produced by random noise may have any of the eight possible range-rates with equal probability. The following probability model is applicable here. Consider k independent trials of an experiment having eight possible mutually exclusive equally probable outcomes. Of these outcomes, 3 specific ones (call them outcomes 1, 2, and 3) are of interest. The probability of obtaining k_1 trials with outcome 1, k_2 trials with outcome 2 and k_3 trials with outcome 3 is given by the multinomial distribution* which, for the present case, may be written

$$P(k_1, k_2, k_3; k) = \frac{k!}{k_1!k_2!k_3!k_4!} \left(\frac{1}{8}\right)^{k_1 + k_2 + k_3} \left(\frac{5}{8}\right)^{k_4} \quad (23)$$

*See ref. 4, Chapter 6.

Here,

$$k_1 + k_2 + k_3 + k_4 = k \tag{24}$$

and k is treated as the parameter of the distribution. The probabilities given by (23) have been computed for all choices of k_1, k_2 and k_3 for $k = 1, 2, \ldots, 10$. There are 1000 of these cases, of which 255 are distinct (there is some redundancy, since k_1, k_2 and k_3 are indistinguishable as far as (23) is concerned).

These multinomial probabilities may be used to compute the conditional probabilities $P(\mathrm{RRCL}|k)$. However, here again the basic mathematical model must be modified, in this case because $k_1 = 0$ does not occur. All cases in which $k_1 = 0$ are, therefore, assigned a probability of 0 and the probabilities for the remaining 715 cases are appropriately adjusted, corresponding to the modification in (18) for the bearing confidence level model. The probabilities $P(\mathrm{RRCL}|k)$ obtained from these modified multinomial probabilities are given in table 4. To compute them, each of the 715 multinomial cases

Table 4. Conditional probabilities $P(\mathrm{RRCL}|k)$ for sequential procedure.

k	RRCL	$P(\mathrm{RRCL}\|k)$	k	RRCL	$P(\mathrm{RRCL}\|k)$
1	0	1	2	0	0.9322
	1	0		1	6.652×10^{-2}
	2	0		2	0
	3	0		3	0
3	0	0.8301	4	0	0.7195
	1	0.1592		1	0.2579
	2	5.888×10^{-3}		2	2.124×10^{-2}
	3	0		3	5.897×10^{-4}
5	0	0.6032	6	0	0.4929
	1	0.3484		1	0.4221
	2	4.572×10^{-2}		2	7.693×10^{-2}
	3	2.886×10^{-3}		3	1.354×10^{-2}
7	0	0.3952	8	0	0.3113
	1	0.4766		1	0.5130
	2	0.1116		2	0.1468
	3	1.653×10^{-2}		3	2.886×10^{-2}
9	0	0.2412	10	0	0.1845
	1	0.5332		1	0.5405
	2	0.1802		2	0.2109
	3	4.477×10^{-2}		3	6.396×10^{-2}

was assigned to the appropriate RRCL (k_1 was considered to be the number of events having the same range-rate as event A; k_2 the number having the next higher range-rate, and k_3 the number having the next lower range-rate), and the probabilities for a given k and RRCL were then summed.

The conditional probabilities in table 4 may be used to compute the over-all probabilities of the range-rate confidence levels as follows:

$$P(\text{RRCL}) \sim \sum_{k=1}^{10} P(\text{RRCL}|k)\, P(k), \qquad (25)$$

where $P(k)$ is given in the last column of table 2. These probabilities are presented in table 5.

Table 5. Range-rate confidence level probabilities for sequential procedure.

RRCL	P(RRCL)
0	0.8913
1	0.1001
2	6.583×10^{-3}
3	4.311×10^{-4}

The products within the summation in (25) are, of course, joint probabilities; i.e., the probability of obtaining a given RRCL and a given k is written

$$P(\text{RRCL},k) = P(\text{RRCL}|k)\, P(k) \qquad (26)$$

Equation 25 is written as an approximation, since values of k greater than 10 have been neglected in the above calculations. Subtracting the sum of the probabilities in the final column of table 2 from 1 gives the probability that k will exceed 10 as 1.676×10^{-4}, so the error introduced by this approximation is slight.

5. FALSE ALARM PROBABILITY

For the purposes of this analysis, a false alarm is defined as a cluster which is produced by random noise and which meets the criteria for entry into PTS. For simplicity, the M threshold will be neglected (this is tantamount to assuming a very high setting for M) and we will assume that all clusters are tested by the confidence level criteria discussed above. The probability of a false alarm P_{FA} is, then, defined as the probability that a cluster, centered on a given event which exceeded the P threshold, meets the confidence level criteria for entry into PTS.

Returning now to the discussion of the decisions made by the computer (*cf.* the paragraph containing table 1 in the Introduction), we see that we can write

$$P_{FA} = Pr\{\text{BCL} + \text{RRCL} \geq 3\} \tag{27}$$

An equivalent statement is that all confidence level combinations on or below the secondary diagonal of the matrix in table 1 lead to acceptance of the cluster (i.e., a false alarm), whereas all combinations above the diagonal lead to rejection of the cluster. We proceed, therefore, to determine the joint probability, denoted $P(\text{BCL},\text{RRCL})$, associated with each of the 16 combinations in table 1. Note that $P(\text{BCL},\text{RRCL}) \neq P(\text{BCL})\ P(\text{RRCL})$, since BCL and RRCL are not independent variables. These are given by

$$\begin{aligned} P(0,0) &= Pr\{\text{RRCL}=0,\ k=1 \text{ or } 2 \text{ or } 3\} \\ &= Pr\{\text{RRCL}=0,\ k=1\} + Pr\{\text{RRCL}=0,\ k=2\} + \\ &\quad Pr\{\text{RRCL}=0,\ k=3\} \end{aligned} \tag{28}$$

$$P(0,1) = Pr\{\text{RRCL}=1,\ k=1 \text{ or } 2 \text{ or } 3\} \tag{29}$$

$$P(1,0) = Pr\{\text{RRCL}=0,\ k=4 \text{ or } 5 \text{ or } 6\} \tag{30}$$

and so on. The joint probabilities involving RRCL and k in these formulas are given by equation 26. The matrix of joint probabilities is given in table 6.

Table 6. Matrix of joint probabilities P(BCL,RRCL) for sequential procedure.

		RRCL				
		0	1	2	3	k
BCL	0	0.7888	5.357×10^{-2}	1.201×10^{-3}	0	1, 2, 3
	1	0.1008	4.430×10^{-2}	4.831×10^{-3}	3.390×10^{-4}	4, 5, 6
	2	1.634×10^{-3}	2.088×10^{-3}	5.113×10^{-4}	8.175×10^{-5}	7, 8
	3	5.043×10^{-5}	1.162×10^{-4}	4.027×10^{-5}	1.041×10^{-5}	

By summing the probabilities on or below the secondary diagonal of this matrix, we obtain

$$P_{FA} = 8.068 \times 10^{-3} \tag{31}$$

The false alarm probability in (31) has been obtained for a specific setting of the T threshold (i.e., 1.8 per cent) and for specific confidence level criteria. This probability will be applied in the next section to obtain an estimate of the false alarm rate for the system, again for these specific conditions.

6. FALSE ALARM RATE

False alarm rate will be discussed here in terms of the mean or expected number of false alarms per ping. A knowledge of the time scale associated with a ping-cycle then permits conversion to false alarms per second, false alarms per convergence zone per second, and so on.

The false alarm probability in (31) is the probability that a cluster, centered on a given event which exceeded the P threshold, meets the confidence level criteria for entry into PTS. If we also knew the expected number of events exceeding P per ping, we could compute the expected number of false alarms per ping. Suppose we assume that P is set to select 1.0 per cent of the events exceeding T. Then, from

(8), the mean number of events exceeding P per unit time is

$$\lambda_P = 0.01\ \lambda_T = 2.88 \times 10^{-3}/t_0 \tag{32}$$

and $\lambda_P t = 40.32$

The probability of entering into TDS per ping exactly k events having a P-bit of 1 is

$$P(k;\ 40.32) = \frac{(40.32)^k}{k!}\ \exp(-40.32). \tag{33}$$

From (31) and (32), the mean number of false alarms per unit time is

$$\lambda_{\mathrm{FA}} = P_{\mathrm{FA}}\lambda_P = 8.068 \times 10^{-3}\lambda_P = 2.32 \times 10^{-5}/t_0 \tag{34}$$

and $\lambda_{\mathrm{FA}}\, t = 0.325.$

The probability of having exactly k false alarms per ping is then

$$P(k;\ 0.325) = \frac{(0.325)^k}{k!}\ \exp(-0.325)$$

$$= 0.723\frac{(0.325)^k}{k!}\ . \tag{35}$$

The expected number of false alarms per ping, which will be denoted N_{FA} and called the false alarm rate, is then

$$N_{\mathrm{FA}} = 0.325. \tag{36}$$

7. REMARKS REGARDING INTERPRETATION

Two basic numbers describing the false alarm performance of the Lorad system (with a coverage of 32° and 3 convergence zones) have been obtained from the above analysis: (1) the false alarm probability in (31); and (2) the false alarm rate in (36). The former is determined by the *T* threshold setting (1.8 per cent in this case), by the criteria used in assigning the confidence levels BCL and RRCL, and by the cluster criterion applied to the confidence level combinations (BCL + RRCL $\geq$ 3 in this case). The latter is determined by these same factors and, in addition, by the *P* threshold setting (1.0 per cent of the events exceeding *T* in this case). The numbers in (31) and (36) apply only for one specific set of conditions; the next section will illustrate the effect on false alarm rate of varying the threshold settings, the confidence level criteria and cluster criterion.

It must be kept in mind (*cf.* footnote, p. 9) that false alarm rate is not a meaningful number by itself. Associated with a given false alarm rate is a detection probability, and these two quantities are generally "trade-off" variables. If one alters the parameters of the system so as to increase the probability of detecting submarines, one should expect to encounter more false alarms per ping. If one alters the parameter values to decrease the rate at which false alarms occur, one should expect to miss (fail to detect) more submarine targets. Therefore, the results presented here must ultimately be coupled with data on the submarine detection performance of the Lorad system in order to arrive at a complete description. This description will provide a powerful tool for predicting and evaluating system performance under a variety of conditions, and for optimizing system performance under specific conditions.

8. CONTROL OF FALSE ALARM RATE

This section considers the effects on false alarm rate of varying the confidence level criteria, the cluster criterion, and threshold settings employed in the system. First, a different set of criteria for assigning BCL is treated; second, several alternate cluster criteria are examined; and third, the dependence of false alarm rate on threshold settings is discussed.

Let us assume that BCL is assigned as follows (*cf.* the procedure described in the Introduction): If the number of events in the cluster-sized region R is 1 or 2, BCL 0 is assigned; if the count is 3, BCL 1 is assigned; if the count is 4, BCL 2 is assigned; and if the count is 5 or greater, BCL 3 is assigned. T is assumed fixed at the 1.8 per cent setting and P at the 1.0 per cent setting. The last column of table 2 yields a new set of probabilities P(BCL) given in table 7. Equations 28, 29, and 30 (and the remaining 13 equations leading to table 6) must be modified so that the proper values of k appear in the right-hand members. The resulting joint probabilities P(BCL,RRCL) are given in table 8. The sum of the probabilities on or below the secondary diagonal of this matrix gives a new false alarm probability P_{FA} of 8.458×10^{-2}; this new value of P_{FA} leads in turn to a new expected false alarm rate N_{FA} of 3.41 per ping. We see that the alternate criteria for assigning BCL increase the false alarm probability and expected false alarm rate by a factor of about 10.5.

Similar effects may be observed if changes are made in the cluster criterion applied to the confidence level combinations. For example, the original BCL criteria with a new cluster criterion BCL + RRCL $\geq$ 2 produce a false alarm probability of 5.52×10^{-2} (using table 6) and a false alarm rate of 2.23. Similarly, the alternate BCL criteria with a new cluster criterion BCL + RRCL $\geq$ 5 produce a false alarm probability of 3.70×10^{-3} (using table 8) and a false alarm rate of 0.149.

The computations in sections 4, 5, and 6 leading to the false alarm rate and the relationship between false alarm rate and false alarm probability may be summarized in the form

$$N_{FA} = 224{,}000\, P\, T\, P_{FA}(T) \qquad (37)$$

P and T denote the threshold settings in probability form (0.01 and 0.018, respectively, for the conditions used to obtain (31) and (36)). $P_{FA}(T)$ is written here to indicate that P_{FA} is determined by T, but is independent of P. It is seen that N_{FA} is directly proportional to P, but is not linearly related to T. We would expect N_{FA} to be more sensitive to changes in T, since an increase in T (lowering of the T threshold setting) would result in an increase in P_{FA}. Table 9 presents false alarm probabilities and false alarm rates for three settings of T. These results were

Table 7. Bearing confidence level probabilities for alternate BCL criteria.

k	BCL	P(BCL)
1,2	0	0.641
3	1	0.204
4	2	9.93×10^{-2}
≥ 5	3	5.57×10^{-2}

Table 8. Matrix of joint probabilities P(BCL, RRCL) for sequential procedure and alternate BCL criteria.

		RRCL				
		0	1	2	3	k
BCL	0	0.6195	2.109×10^{-2}	0	0	1,2
	1	0.1693	3.248×10^{-2}	1.203×10^{-3}	0	3
	2	7.145×10^{-2}	2.561×10^{-2}	2.109×10^{-3}	5.856×10^{-5}	4
	3	2.107×10^{-2}	2.089×10^{-2}	3.273×10^{-3}	3.726×10^{-4}	≥ 5

Table 9. P_{FA} and N_{FA} versus T threshold setting.

T	P_{FA}	N_{FA}
0.01	1.171×10^{-3}	$2.62P$
0.018	8.068×10^{-3}	$32.5P$
0.025	2.565×10^{-2}	$143.6P$

obtained using the original confidence level and cluster criteria (i.e., those used to obtain equations 31 and 36).

9. TIME INTERVALS BETWEEN EVENTS

Equation 1 gives the probability of finding exactly k events in an interval of length t. Then

$$P(o;\lambda t) = \exp(-\lambda t) \tag{38}$$

is the probability of finding no events in an interval of length t. We may state this in another equivalent way: equation 38 gives the probability that the interval between two events exceeds t.

At this point, it would be useful to review the situation regarding the occurrence of events in time. Either 0 or 1 events can occur in a time interval t_o (corresponding to a 4-yard range interval) in a single beam, and we have assumed that consecutive intervals of length t_o are independent. From the above discussion, $P[0;\ \lambda(n-1)t_o]$ is the probability that the interval between events exceeds $(n-1)t_o$ and $P(0;\ \lambda nt_o)$ is the probability that the interval between events exceeds nt_o. Thus, the probability $P(nt_o)$ that the interval between events is nt_o is given by

$$\begin{aligned} P(nt_o) &= P[0;\ \lambda(n-1)t_o] - P(0;\ \lambda nt_o) \\ &= \exp[-\lambda(n-1)t_o] - \exp(-\lambda nt_o) \\ &= \exp(-\lambda nt_o)[\exp(\lambda t_o) - 1],\ n \geq 1. \end{aligned} \tag{39}$$

For a single beam, $\lambda_o = 0.018/t_o$ and from (39)

$$P_o(nt_o) = 0.018\ \exp(-0.018n),\ n \geq 1. \tag{40}$$

For 16 beams, $\lambda_T = 0.288/t_o$ and

$$P_T(nt_o) = 0.334 \exp(-0.288n), \; n \geq 1. \tag{41}$$

Equation 41 gives the probability distribution of the time intervals between events exceeding the T threshold. Equations 40 and 41 are plotted in figure 3.

It is interesting to note that (39) may also be obtained by approaching the problem from a more general point of view. It is known[6] that if the time interval between randomly occurring events is allowed to take on all values (i.e., if t is a continuous variable rather than a discrete one), the probability density function of the intervals is

$$p(t) = \lambda \exp(-\lambda t). \tag{42}$$

This is called the exponential density function and the moments are

$$\mu_j' = j!/\lambda^j \tag{43}$$

$$\mu = 1/\lambda \tag{44}$$

$$\sigma^2 = 1/\lambda^2 \tag{45}$$

We may now obtain (39) by integrating (42)

$$P(nt_o) = \int_{(n-1)t_o}^{nt_o} p(t)dt$$

$$= \exp(-\lambda n t_o)[\exp(\lambda t_o) - 1], \; n \geq 1.$$

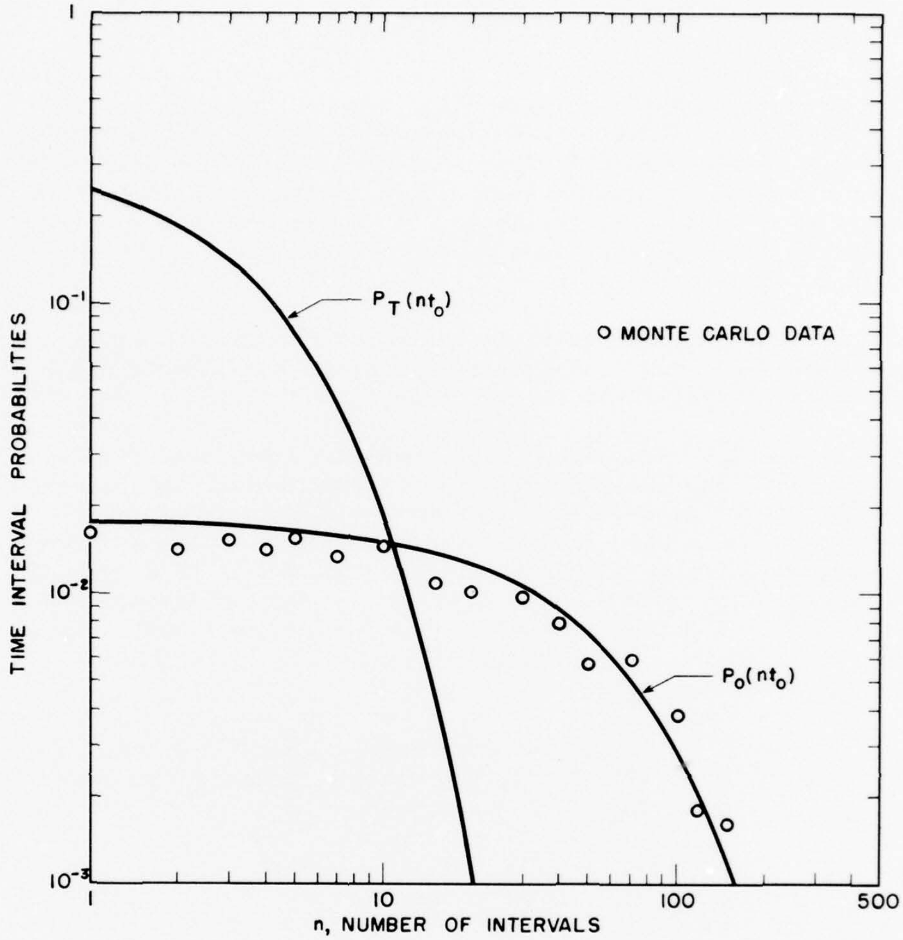

Figure 3. Time-interval probabilities.

CONCLUSIONS

1. Conventional probability distributions and techniques applicable to discrete random variables may be employed to provide a thorough analytic treatment of automatic detection processes in the Lorad system. This treatment encompasses the use of thresholds for amplitude selection and computer programs for cluster (i.e., possible targets based on single-ping criteria) selection.

2. If the threshold settings and cluster selection criteria are specified, the false alarm probability and false alarm rate of the system can be predicted.

3. Since these techniques have been developed in a form which makes the analysis of a system essentially independent of the amplitude distributions involved, they should be applicable to a wide variety of modern search systems. Both analog and digital processes are susceptible to this type of analysis. The only basic limitation is that the system must process its information in a sampled form; events must be presented to the automatic detection portion of the system at discrete times.

4. The analytic techniques developed above should apply equally well to automatic target classification processes.

RECOMMENDATIONS

1. Apply the techniques to other sonar systems: SQS-23 modified for use with Small Ship Combat Data System (SSCDS); SQS-26.

2. Apply the methods to target classification processes in Lorad.

3. Develop detection probabilities for the Lorad system and couple these with false alarm characteristics derived in this report.

APPENDIX A: ALTERNATE PROCEDURE FOR ASSIGNING RRCL

A sequential procedure for assigning RRCL was described in the Introduction and treated analytically in section 4 above. This procedure examines the minimum number of range-rate channels, since it assigns RRCL as soon as it encounters a channel containing two or more events. Some opposition has arisen to this procedure because it may fail to examine a channel which has many more events in it than the channel upon which RRCL was based; this would result in assignment of an RRCL which is too small and, in addition, would lead to an incorrect decision concerning the appropriate range-rate to be attached to the cluster for display.

The most obvious alternate procedure, which avoids these objections entirely, is to examine all three range-rate channels of interest for each event and to assign RRCL on the basis of the channel containing the largest count. There are, of course, other procedures which lie between these two in terms of the extent to which they circumvent the stated objections and in terms of the amount of computation time required, but they need not be considered here.

The most important question which arises is: how much does the largest count procedure raise the probability of a false alarm? Clearly it will increase P_{FA}, since the RRCL assigned by the largest count procedure is greater than or equal to that assigned by the sequential procedure. The computation of P_{FA} for the largest count procedure has been carried out, retaining the same T threshold setting and BCL criteria used to obtain (31). The basic multinomial model still applies, since we are still interested in three of the eight range-rate channels. Tables 4, 5, and 6 are replaced by tables 10, 11, and 12.

Summing the probabilities on or below the secondary diagonal of the matrix (see table 12) in accordance with (27), we obtain

$$P_{FA} = 8.186 \times 10^{-3} \qquad (46)$$

for the largest count procedure. Comparison of (46) and (31) indicates that P_{FA} is, for practical purposes, unaffected by procedural modifications of the type considered here.

Table 10. Conditional probabilities $P(\mathrm{RRCL}|k)$ for largest count procedure.

k	RRCL	$P(\mathrm{RRCL}\|k)$	k	RRCL	$P(\mathrm{RRCL}\|k)$
1	0	1	2	0	0.9322
	1	0		1	6.652×10^{-2}
	2	0		2	0
	3	0		3	0
3	0	0.8301	4	0	0.7195
	1	0.1592		1	0.2579
	2	5.888×10^{-3}		2	2.124×10^{-2}
	3	0		3	5.897×10^{-4}
5	0	0.6032	6	0	0.4929
	1	0.3472		1	0.4165
	2	4.698×10^{-2}		2	8.232×10^{-2}
	3	2.886×10^{-3}		3	1.374×10^{-2}
7	0	0.3952	8	0	0.3113
	1	0.4620		1	0.4840
	2	0.1251		2	0.1721
	3	1.769×10^{-2}		3	3.255×10^{-2}
9	0	0.2412	10	0	0.1845
	1	0.4845		1	0.4680
	2	0.2203		2	0.2664
	3	5.344×10^{-2}		3	8.098×10^{-2}

Table 11. Range-rate confidence level probabilities for largest count procedure.

RRCL	$P(\mathrm{RRCL})$
0	0.8913
1	9.987×10^{-2}
2	6.777×10^{-3}
3	4.429×10^{-4}

Table 12. Matrix of joint probabilities $P(\mathrm{BCL}, \mathrm{RRCL})$ for largest count procedure.

		RRCL 0	RRCL 1	RRCL 2	RRCL 3	k
BCL	0	0.7888	5.357×10^{-2}	1.201×10^{-3}	0	1, 2, 3
	1	0.1008	4.419×10^{-2}	4.947×10^{-3}	3.415×10^{-4}	4, 5, 6
	2	1.634×10^{-3}	2.012×10^{-3}	5.795×10^{-4}	8.888×10^{-5}	7, 8
	3	5.043×10^{-5}	1.048×10^{-4}	4.952×10^{-5}	1.259×10^{-5}	≥ 9

APPENDIX B: MONTE CARLO EXPERIMENTS

The following Monte Carlo experiments were performed on the NTDS unit computer to check some of the distributions employed in the above analysis: (1) generate a large set of Rayleigh-distributed samples; (2) threshold the samples to select 1.8 per cent; (3) check the Poisson distribution in (16); and (4) check the exponential distribution in (40).

1. AMPLITUDE DISTRIBUTION OF CORRELATION VALUES

It has been demonstrated above that a thorough analytical treatment of the detection and false alarm performance of a system such as Lorad (in fact, any system which has as its processed outputs sequences of amplitudes occurring at discrete times) can be carried out without detailed knowledge of the probability distribution of these amplitudes. For the purposes of Monte Carlo simulation, however, some amplitude distribution had to be selected. The correlation values at the input to the T threshold were assumed to be Rayleigh-distributed, an assumption which appears to agree reasonably well with experimental data.

The Rayleigh probability density function will be written in the form

$$p_1(r) = \frac{2r}{\theta} \exp(-r^2/\theta),\ r \geq 0. \tag{47}$$

The moments of this distribution are

$$\mu'_j = \theta^{j/2}\Gamma\left(\frac{j+2}{\theta}\right) = \frac{j\theta}{2}\mu'_{j-2} \tag{48}$$

$$\mu = \tfrac{1}{2}(\pi\theta)^{\frac{1}{2}} \tag{49}$$

$$\mu'_2 = \theta \tag{50}$$

$$\sigma^2 = \theta\left(\frac{4-\pi}{4}\right) \tag{51}$$

$\theta^{\frac{1}{2}}$ is, then the RMS level of the threshold inputs.

2. GENERATION OF RAYLEIGH SAMPLES

Programs to generate samples from various distributions have been written for the NTDS unit computer. Large samples from the following four distributions have been successfully generated to date: the rectangular density function (samples from this distribution are called uniform deviates)

$$p_2(U) = \begin{cases} 1, & 0 \leq U \leq 1 \\ 0, & \text{elsewhere} \end{cases} ; \tag{52}$$

the standard Gaussian density function (samples are called normal deviates) in (11); the Rayleigh density function in (47) with $\theta = 2$

$$p_1(r) = r \exp(-r^2/2); \tag{53}$$

and the exponential density function in (42) with $\lambda = \frac{1}{2}$

$$p(t) = \tfrac{1}{2} \exp(-t/2) \tag{54}$$

The details concerning the generation techniques employed will be reported separately. The techniques will be described very briefly here.

The rectangular distribution was generated first, using the "congruential" method.[7] Then, normal deviates were generated by adding ten of the rectangularly-distributed numbers (the central limit theorem approach).[8] Rayleigh-distributed numbers were generated by taking two independent sets of normal deviates, the members of which are denoted x_1 and x_2 and computing $r = \left(x_1^2 + x_2^2\right)^{\frac{1}{2}}$. And, finally, exponentially-distributed numbers with $\lambda = \frac{1}{2}$ were generated by squaring the Rayleigh-distributed numbers.

It will be noted that the Rayleigh-samples are obtained through a series of successive approximations. Thus, these samples will not have a perfect Rayleigh distribution; in particular, the worst deviations from the theoretical Rayleigh density function are to be expected for large values of r, since normal deviates generated in the manner described do not

agree very well with (11) for large values of x. There are other normal-deviate-generating techniques which result in a better fit of the samples to (11).[8] Some of these will be programmed and checked in the near future.

The theoretical Rayleigh density function in (53) is plotted in figure 4, along with amplitude distribution data obtained from 10,000 computer-generated samples. The theoretical moments for (53) are given by (49) and (51) with $\theta = 2$

$$\mu = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} = 1.253 \tag{55}$$

$$\sigma^2 = \left(\frac{4-\pi}{2}\right) = 0.4292 \tag{56}$$

whereas the empirical moments, denoted $\bar{r}$ and s^2 for the 10,000 samples were found to be

$$\bar{r} = 1.252 \tag{57}$$

$$s^2 = 0.419. \tag{58}$$

It will be seen that the agreement is relatively good, and a χ^2 test indicates that the agreement is statistically acceptable at the 5 per cent level of significance.

3. THRESHOLDED RAYLEIGH SAMPLES

The threshold value $r_c = 2.834$ should select 1.8 per cent of the Rayleigh samples; that is,

$$\int_{r_c=2.834}^{\infty} r \exp(-r^2/2)dr = 0.018 \tag{59}$$

The "tail" of the Rayleigh density function for $r \geq r_c$ is plotted in figure 5, along with amplitude distribution data for 10,000 computer-generated samples which exceeded r_c. Here again, the agreement is satisfactory, but the predicted deviation from the theoretical density function for large

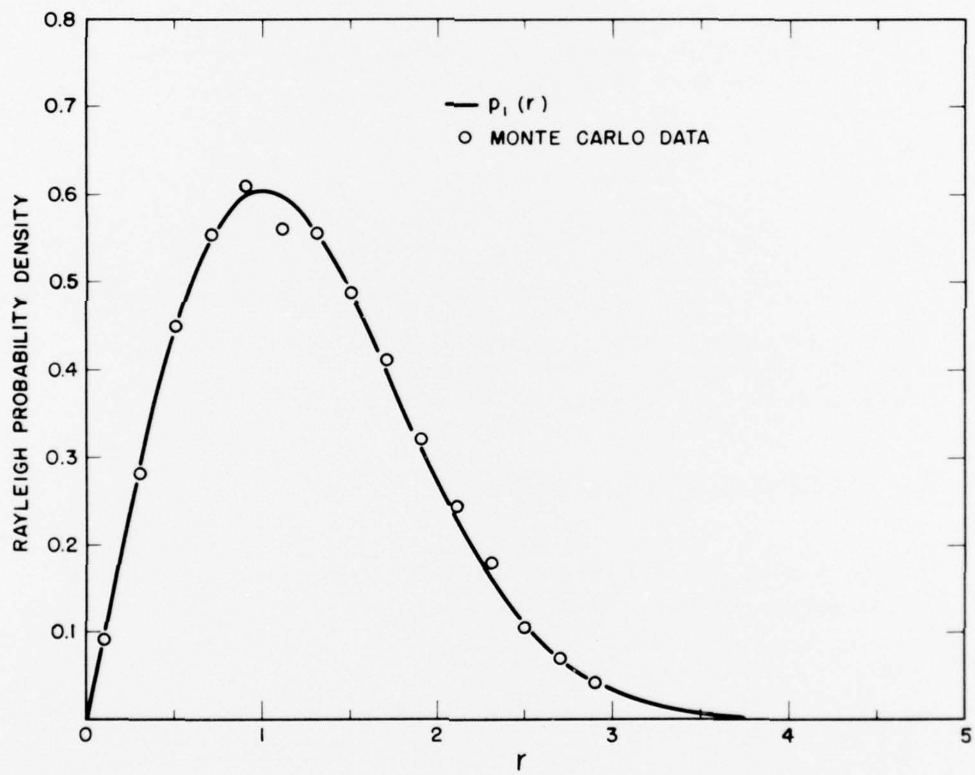

Figure 4. Rayleigh probability density function.

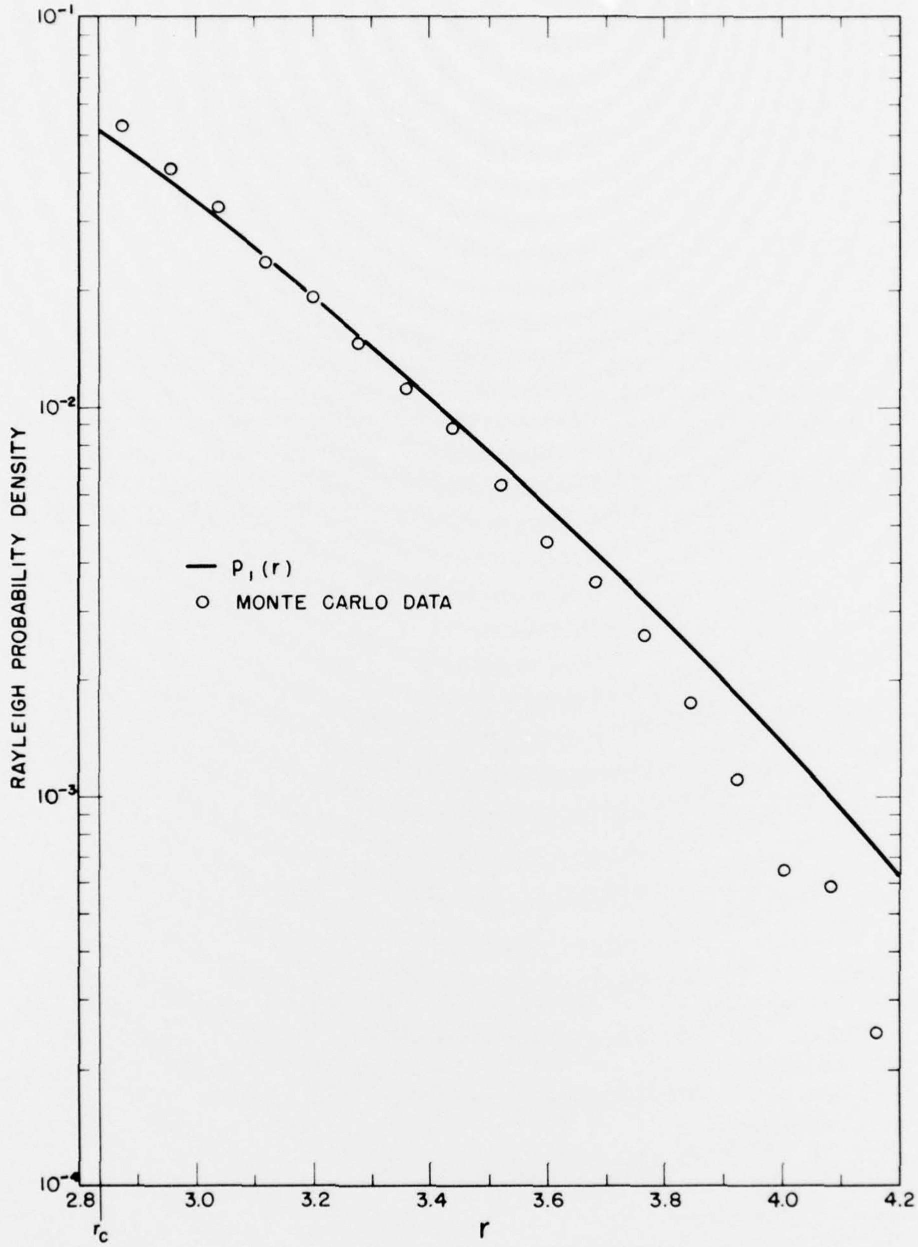

Figure 5. Tail of Rayleigh probability density function.

values of r is observed. In particular, the Monte Carlo data fall rather consistently below the theoretical curve. It was found, in fact, that the threshold value of 2.834 selected only about 1.59 per cent of the Rayleigh samples instead of 1.8 per cent. The effects of this will be seen in the next two sections.

4. SPATIAL CLUSTER PROBABILITY

The Poisson distribution in (16) was checked by generating 10,000 sets of 27 Rayleigh samples and counting the number of samples exceeding threshold in each set. 27 is the number of 4-yard range intervals in the 108-yard interval corresponding to a cluster in a single beam. This procedure yielded 10,000 Poisson samples, the distribution for which is tabulated in table 2 and plotted in figure 2. The theoretical moments are

$$\mu = \sigma^2 = 0.486 \tag{60}$$

whereas the moments for the 10,000 samples were found to be

$$\bar{k} = 0.429 \tag{61}$$

$$s^2 = 0.410 \tag{62}$$

In this case, the agreement is sufficiently good to bear out the use of a Poisson distribution. The discrepancy between the data and (16) is attributable to the deviation between the amplitude distribution of the thresholded Rayleigh samples and the theoretical density function. In support of this rationalization, it is noted that if the 1.8 per cent figure used in (6) were replaced by 1.59 per cent, the constant $\lambda_c t = 0.486$ in (16) and (60) would be replaced by 0.429, which agrees with (61).

5. TIME INTERVALS BETWEEN EVENTS

To check (40), Rayleigh samples were generated until 10,000 had exceeded threshold. In each case, the number of samples

generated in order to get a sample that exceeded threshold was counted. The distribution of intervals is plotted as open circles in figure 3, for comparison with the curve for $P_o(nt_o)$. The theoretical moments for (40) are given approximately by the theoretical moments of (42) with $\lambda = 0.018$

$$\mu = \sigma = 55.6 \tag{63}$$

whereas the moments for the 10,000 samples were found to be

$$\bar{n} = 63.09 \tag{64}$$

$$s = 54.41. \tag{65}$$

Although the agreement between the data and the theoretical distribution is relatively good here, we note that if 1.8 per cent is again replaced by 1.59 per cent, the number 55.6 in (63) is replaced by 62.9, which agrees well with (64).

[redacted]

APPENDIX C: GLOSSARY

The following is a glossary of the basic symbols, abbreviations, and notations employed in this report, in approximate order of appearance.

T	Analog threshold which selects events for entry into TDS (also the probability that an event will exceed T)
TDS	Temporary data storage list in the computer
P	Analog threshold which selects the largest events exceeding T for further analysis (also the probability that an event will exceed P)
A	A particular event which exceeds P
M	Digital threshold which selects the largest events exceeding P for retention and display on the basis of amplitude alone
PTS	Possible target storage list in the computer
BCL	Bearing confidence level
RRCL	Range-rate confidence level
$P(\ )$	General notation for the probability distribution of a discrete variable or the probability of a particular value of the variable
$Pr\{\ \}$	General notation for the probability of an occurrence, the occurrence being described inside the brackets
k	Number of events - an integer
S	Number of events observed in a particular case
λ	Parameter of a Poisson distribution (λ_o, λ_T, λ_P, and λ_{FA} are special cases)
t	A time interval
μ'_j	jth moment about zero of a distribution
μ_j	jth central moment (about μ) of a distribution
μ	Mean of a distribution ($\mu = \mu'_1$)

[redacted]

σ^2	Variance of a distribution ($\sigma^2 = \mu_2$) (σ = standard deviation)
t_o	5-millisecond time interval corresponding to a 4-yard range increment
x	Random variable having a standard ($\mu = 0$ and $\sigma = 1$) Gaussian (normal) probability density function
$\phi(x)$	The standard ($\mu = 0$ and $\sigma = 1$) Gaussian (normal) probability density function
$\Phi(x)$	The cumulative standard Gaussian distribution function
R	A region having a range extent of 108 yards and a bearing extent of four 2° beams (cluster-sized region)
P_{FA}	False alarm probability
N_{FA}	False alarm rate
n	Number of intervals of duration t_o - an integer
$p(\ )$	General notation for a probability density function
r	Random variable having a Rayleigh probability density function
θ	Parameter of a Rayleigh density function
U	Random variable having a rectangular density function between 0 and 1
$\bar{r}$	The bar is general notation for the sample (empirical) mean of a distribution
s^2	Sample variance of a distribution

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